MAS 2103-001

Name: Practice Exam 1

Please be sure to read each question carefully and show ALL of your work.

Problem 1: Find the solution set of the following system of equations:

$$\begin{aligned} x_1 + 3x_2 &= 4 \\ -2x_1 - 5x_2 &= -7 \\ x_1 + 2x_2 &= 2 \end{aligned}$$

Problem 2: Determine if the set of vectors $\{v_1, v_2, v_3\}$ is a linearly independent set. If they are **not** linearly independent, find scalars c_1, c_2, and c_3 (not all zero) such that $c_1v_1 + c_2v_2 + c_3v_3 = 0$.

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}, \quad v_3 = \begin{bmatrix} -2 \\ 1 \\ -0 \end{bmatrix}$$

Problem 3: Solve the matrix equation $A\mathbf{x} = \mathbf{b}$, where A and $\mathbf{b}$ are given below. Express your solutions in parametric vector form.

$$A = \begin{bmatrix} 2 & 1 & 1 & 3 \\ -1 & 2 & 0 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Problem 4: Let A be the linear transformation defined by the matrix below. Find the domain and codomain of A.

$$A = \begin{bmatrix} 1 & -1 & 1 & 1 & 2 \\ 0 & 0 & 2 & -3 & 7 \\ 4 & -3 & 0 & 1 & 0 \end{bmatrix}$$

Problem 5: For the following two problems, determine the matrix that represents the given linear transformation T.

a) T is a linear transformation such that $T(\mathbf{e_1}) = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$ and $T(\mathbf{e_2}) = \begin{bmatrix} -1 \\ 0 \\ 9 \end{bmatrix}$.

b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation that **reflects** vectors across the x-axis.

Problem 6: Calculate the inverse of each of the following matrices.

a) $A = \begin{bmatrix} 2 & 3 \\ -1 & -1 \end{bmatrix}$

b) $B = \begin{bmatrix} 3 & 3 & 1 \\ 0 & 1 & 2 \\ -1 & -1 & 0 \end{bmatrix}$

Problem 7: Calculate the following matrix determinants.

a) $\begin{vmatrix} 2 & 0 & 1 \\ 2 & 3 & 1 \\ 0 & 4 & -2 \end{vmatrix}$

b) $\begin{vmatrix} 1 & 5 & 2 & -4 \\ 0 & -2 & 3 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 \end{vmatrix}$

(*Note:* You may use the rule for the determinant of a triangular matrix. If you forget the rule, you can calculate the determinant by hand, but just make sure to cleverly correct row/column for the cofactor expansion.)

Solutions:

1. No solution.
2. Not linearly independent. One possible linear combination is $2v_1 - v_2 - v_3 = 0$.
3. Sorry for the fractions, but

$$\mathbf{x} = \begin{bmatrix} 3/5 \\ -1/5 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -2/5 \\ -1/5 \\ 1 \\ 0 \end{bmatrix} + r \begin{bmatrix} -1 \\ -1 \\ 0 \\ 1 \end{bmatrix}.$$

4. $A : \mathbb{R}^5(\text{domain}) \to \mathbb{R}^3(\text{codomain})$
5. a) $\begin{bmatrix} 1 & -1 \\ 3 & 0 \\ 1 & 9 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
6. a) $A^{-1} = \begin{bmatrix} -1 & -3 \\ 1 & 2 \end{bmatrix}$ b) $B^{-1} = \begin{bmatrix} 2 & -1 & 5 \\ -2 & 1 & -6 \\ 1 & 0 & 3 \end{bmatrix}$
7. a) -12 b) -12