MAS 2103-001

Name: Practice Exam 2

Please be sure to read each question carefully and show ALL of your work.

Problem 1: Determine which of the following sets are subspaces of the given vector space and **justify** your response.

a) Let $W = \left\{ \begin{bmatrix} a+b \\ a-b \\ 2 \end{bmatrix} : a \text{ and } b \text{ are real numbers.} \right\}$. Is W a subspace of $\mathbb{R}^3$?

b) Recall that $\mathbb{P}_n$ is the set of all polynomials of degree less than or equal to n. Is $\mathbb{P}_2$ a subspace of $\mathbb{P}_3$?

Problem 2: Find a matrix A such that the given set is Col A.

$$\left\{ \begin{bmatrix} 8r - s + t \\ -3s \\ r - t \\ 2r + 2s + 2t \end{bmatrix} : r, s, t \text{ are real numbers.} \right\}$$

Problem 3: For the following matrix A, find an explicit description of Nul A by listing vectors that span the null space.

$$A = \begin{bmatrix} 1 & -1 & 1 & 3 \\ 0 & -2 & 4 & 0 \end{bmatrix}$$

Problem 4: For the following two problems, consider the below matrix A.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 3 & -6 \\ 0 & -4 & 4 \end{bmatrix}$$

a) Find a basis for Nul A and calculate the nullity of A.

b) Find a basis for Col A and calculate the rank of A.

Problem 4: For the following three problems, consider the following matrix A.

$$A = \begin{bmatrix} 8 & 10 \\ -5 & -7 \end{bmatrix}$$

a) Calculate the eigenvalues of A.

b) For each eigenvalue of A, find a basis for the corresponding eigenspace.

c) Diagonalize the matrix A by finding a matrix P And a diagonal matrix D such that $A = PDP^{-1}$.

Problem 5: For the following three problems, determine whether the given expression is well-defined. If it is, determine the value. If it is not, explain why.

$$\mathbf{u} = \begin{bmatrix} 3 \\ -2 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$$

a) $\mathbf{u} \cdot \mathbf{u} - 2\,(\mathbf{u} \cdot \mathbf{v}) + \mathbf{v} \cdot \mathbf{v}$

b) $\dfrac{||\mathbf{v}||}{||\mathbf{u}||}\mathbf{u}$

c) $\sqrt{(\mathbf{v} - \mathbf{u})^2 + (\mathbf{u} - \mathbf{v})^2}$

Solutions:

1. a) No, none of the three subspace axioms are satisfied
   b) Yes, all three subspace axioms are satisfied.

2. One possible solution: $A = \begin{bmatrix} 8 & -1 & 1 \\ 0 & -3 & 0 \\ 1 & 0 & -1 \\ 2 & 2 & 2 \end{bmatrix}$

3. One possible choice: $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

4. a) Possible basis: $\left\{ \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} \right\}$, nullity = 1.
   b) Possible basis: $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix} \right\}$, rank = 2.

5. a) $\lambda_1 = 3$, $\lambda_2 = -2$
   b) Possible basis for $\mathbb{E}_3 : \left\{ \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}$ Possible basis for $\mathbb{E}_{-2} : \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$
   c) $P = \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$, $D = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}$

6. a) 65 b) $\begin{bmatrix} 3\sqrt{2} \\ -2\sqrt{2} \end{bmatrix}$ c) Not possible– cannot square a vector